

B.Tech. 4th Semester F-Scheme
(Common for All Branches) Examination,
May-2019

MATHEMATICS-III
Paper-Math-201-F

Time allowed : 3 hours [Maximum marks : 100]

Note : Attempt five questions in total selecting one question from each section. Question No. 1 is compulsory.

1. (a) Find the value of a_n in the Fourier series of $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$,
- (b) Separate into real and imaginary parts $\text{Log}(4+3i)$.
- (c) Define residue. Write statement of Cauchy's residue theorem.
- (d) Solve the following LPP graphically :
Minimize $Z = 3x + 2y$
subject to the constraints
 $5x + y \geq 10, x + y \geq 6, x + 4y \geq 12, x, y \geq 0$.

Section-A

2. (a) Find the Fourier series expansion for :

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

- (b) Express $f(x) = x$ as a half-range sine series in $0 < x < 2$.
3. Solve the partial differential equation :

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}, \quad u > 0, t > 0$$

subject to the conditions :

- (a) $u(0, t) = 0, t > 0$
- (b) $u(x, 0) = e^{-x}, x > 0$
- (c) u and $\frac{\partial u}{\partial x} \rightarrow 0$ as $x \rightarrow \infty$

Section-B

4. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, even though C-R equations are satisfied there at.
- (b) Determine the analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$.
5. (a) Evaluate $\oint_C |z|^2 dz$, around the square with vertices at $(0,0), (1,0), (1,1)$ and $(0,1)$.

- (b) Use Cauchy's integral formula to evaluate

$$\oint_C \frac{2^{2z}}{(z+1)^4} dz, \text{ where } C \text{ is the circle } |z|=2.$$

Section-C

6. (a) Expand $\frac{1}{z^2 - 3z + 2}$ in the region :

(i) $1 < |z| < 2$

(ii) $|z| < 2$

- (b) Evaluate $\oint_C \frac{z^3}{(z-1)^2(z-2)(z-3)} dz$.

Where C is the circle $|z| = \frac{5}{2}$. By using residue theorem.

7. (a) A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and the variance of the number of successes.
- (b) Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six ?

Section-D

8. (a) The sales in a super market during a week are given below. Test the hypothesis that the sales do not depend on the day of the week at 5% level of significance :

| Days | : | Mon | Tues | Wed | Thus | Fri | Sat |
|---------------------------|---|-----|------|-----|------|-----|-----|
| Sales (in 1000 Rs.) | : | 65 | 54 | 60 | 56 | 71 | 84 |

- (b) A sample of 18 items has a mean 24 units and standard deviation 3 units. Test the hypothesis that it is a random sample from a normal population with mean 27 units.
9. Using dual simplex method

$$\text{Maximize } z = -3x_1 - 2x_2$$

$$\text{Subject to } x_1 + x_2 \geq 1,$$

$$x_1 + x_2 \leq 7,$$

$$x_1 + 2x_2 \geq 10,$$

$$x_2 \leq 3,$$

$$x_1, x_2 \geq 0$$